

STOCHASTIC EXCITATION OF STELLAR OSCILLATIONS

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Abstract

Excitation of solar oscillations is attributed to turbulent motions in the solar convective zone. It is also currently believed that oscillations of low massive stars ($M < 2 M_{\odot}$) - which possess an upper convective zone - are stochastically excited by turbulent convection in their outer layers.

A recent theoretical work (Samadi and Goupil, 2001; Samadi et al., 2001) supplements and reinforces this theory. This allows the use of any available model of turbulence and emphasizes some recent unsolved problems which are brought up by these new theoretical developments.

1 Introduction

The excitation of the solar oscillations results from the action of the Reynolds tensor and of the turbulent fluctuations of entropy; the latter comes from a heat exchange between the oscillations and the turbulent elements. The nature and the properties of the source term of excitation due to the Reynolds tensor are well established (Goldreich and Keeley, 1977), although the evaluation of this term remained crudely approximated. On the other hand, the source of excitation which takes its origin in the fluctuations of the entropy, had given rise to major controversies (e.g. Goldreich et al., 1994, GMK hereafter).

In Samadi and Goupil (2001, Paper I hereafter), inconsistencies in the available theories were removed which led to a formulation which generalizes results from previous works, and is built so as to enable consistent investigations of various possible spatial and temporal spectra of stellar turbulent convection. It also shows that the actual entropy source term results from the advection of the turbulent fluctuations of entropy by the turbulent motions. These results are summarized in Section 2 and Section 3.

The unavoidable free parameters introduced in the formulation are adjusted in order to obtain the best fit to the solar seismic observations. In a second step, the comparison between the computed amplitudes and the observations as well as the use of theoretical arguments allow us to determine the ingredients in the theory which are still defective. Section 4 then lists some unsolved problems which are brought up by our improvement of the theoretical approach in Paper I. Final comments conclude about elements of the theory which still remain to be developed.

2 Theory of the stochastic excitation

The stochastic excitation mechanism results from a forcing of the stellar material by the turbulent motions and by a heat exchange due to the turbulent fluctuations of the entropy; in other words, the acoustic power generated by the turbulence excites resonant modes of the stellar cavity (oscillations).

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In the presence of turbulence one shows that the radial oscillation velocity \vec{v}_{osc} obeys the inhomogeneous wave equation (see Paper I for details):

$$\rho_0 \left(\frac{\partial^2}{\partial t^2} - \vec{L} \right) [\vec{v}_{osc}] + \vec{\mathcal{D}}(\vec{v}_{osc}) = \frac{\partial}{\partial t} (\vec{f}_t + \vec{\nabla} h_t) \quad (2.1)$$

with

$$\vec{f}_t = -\vec{\nabla} : (\rho_0 \vec{u} \vec{u}) \quad (2.2)$$

$$\frac{\partial}{\partial t} \vec{\nabla} h_t = -\vec{\nabla} \left(\alpha_s \frac{d}{dt} \delta s_t - \vec{\nabla} \cdot (\alpha_s \vec{u} s_t) + s_t \vec{u} \cdot \vec{\nabla} \alpha_s \right) \quad (2.3)$$

the turbulent Reynolds stress and the source term due to the turbulent entropy fluctuations respectively. The expression for the wave operator \vec{L} is given in Paper I. In Eqs.(2.1 , 2.2, 2.3), ρ , s , c_s , \vec{g} and \vec{u} respectively denote density, entropy, sound speed, gravitational acceleration and velocity of the turbulent elements; the subscript 0 refers to average mean quantities.

The first term in the RHS of Eq.(2.3) is due to the Lagrangian entropy fluctuation δs_t . The last two terms are due to the buoyancy force associated with the Eulerian entropy fluctuations s_t . These terms are found to contribute to the excitation as much as the Reynolds source term. Other source terms in the RHS of Eq.(2.1) are found negligible in Paper I and are discarded here.

The operator $\vec{\mathcal{D}}$ in Eq.(2.1) is responsible for generating both a dynamical damping and a shift of the oscillation frequency due to the action of the turbulent elements. Contribution of the operator $\vec{\mathcal{D}}$ to the damping is expected to be small compared to those of the other damping processes. and is assumed to be included in the global damping rate η which takes into account all damping processes.

Assuming no turbulence ($\vec{u} = 0$) the velocity field is simply related to the pulsational radial displacement as $\vec{v} = \frac{d}{dt} \vec{\xi}_r(\vec{r}, t)$ where $\vec{\xi}_r(\vec{r}, t)$ is the undriven pulsational radial displacement in the absence of turbulence. Eigensolutions of Eq.(2.1) complemented with boundary conditions, can be written, in the absence of turbulence, in terms of the displacement $\vec{\xi}_r(\vec{r}, t) = \vec{\xi}_r(\vec{r}) e^{-i\omega_0 t}$ where ω_0 is the oscillation frequency and $\vec{\xi}_r(\vec{r})$ is the adiabatic (real) displacement eigenvector which satisfies $\vec{L}(\vec{\xi}_r(\vec{r})) = -\omega_0^2 \vec{\xi}_r(\vec{r})$.

When turbulence is present, the pulsational displacement and velocity are written in terms of the above adiabatic solution $\vec{\xi}_r(\vec{r}, t)$ and an instantaneous amplitude $A(t)$. Accordingly

$$\delta \vec{r}_{osc} = \frac{1}{2} \left(A(t) \vec{\xi}_r(\vec{r}) e^{-i\omega_0 t} + cc \right) \quad \text{and} \quad \vec{v}_{osc} = \frac{1}{2} (-i\omega_0 A(t) \vec{\xi}_r(\vec{r}) e^{-i\omega_0 t} + cc) \quad (2.4)$$

where cc means complex conjugate. The mode energy, averaged over a time scale smaller than the mode lifetime η^{-1} and larger than the eddies time scale, is given by

$$E = \int_0^M dm \langle v_{osc}^2 \rangle = \frac{1}{2} \langle |A|^2 \rangle I \omega_0^2 \quad (2.5)$$

where $I \equiv \int_V \rho_0 d^3x (\vec{\xi}_r^* \cdot \vec{\xi}_r)$ is the mode inertia and $\langle |A|^2 \rangle$ is the mean square amplitude. Finally a general expression for the power, P , going into each mode is established in Paper I:

$$P \equiv \frac{dE}{dt} = \eta \langle A^2 \rangle I \omega_0^2 \quad \text{with} \quad \langle |A|^2 \rangle = \frac{1}{8\eta(\omega_0 I)^2} (C_R^2 + C_S^2) \quad (2.6)$$

where C_R^2 and C_S^2 are the turbulent Reynolds stress and the entropy contributions respectively given by:

$$C_R^2 = \int d^3x_0 \int_{-\infty}^{+\infty} d\tau e^{-i\omega_0 \tau} \int d^3r \langle (\rho_0 u_j u_i \nabla^j \xi_r^i)_1 (\rho_0 u_j u_i \nabla^j \xi_r^i)_2 \rangle \quad (2.7)$$

$$C_S^2 = \int d^3x_0 \int_{-\infty}^{+\infty} d\tau e^{-i\omega_0 \tau} \int d^3r \langle (h_t \vec{\nabla} \cdot \vec{\xi}_r)_1 (h_t \vec{\nabla} \cdot \vec{\xi}_r)_2 \rangle \quad (2.8)$$

where \vec{x}_0 is the average position where the power is evaluated whereas \vec{r} and τ are related to the local turbulence. Subscripts 1 and 2 refer to the spatial and temporal positions $[\vec{x}_0 - \frac{\vec{r}}{2}, -\frac{\tau}{2}]$ and $[\vec{x}_0 + \frac{\vec{r}}{2}, \frac{\tau}{2}]$ respectively.

2.1 Reynolds stress contribution

It is shown that the terms $(\rho_0 \nabla_1^i \xi_r^j)$ and $(\rho_0 \nabla_2^i \xi_r^j)$ in Eq.(2.7) do not change over the length scale of the eddies. Consequently, in Eq.(2.7), integrations over τ and r only involve the phase term $e^{-i\omega_0\tau}$ and the fourth-order velocity correlations $\langle u'_i u'_j u''_l u''_m \rangle$ where $\vec{u}' = \vec{u}(\vec{x}_0 - \vec{r}/2, -\tau/2)$ and $\vec{u}'' = \vec{u}(\vec{x}_0 + \vec{r}/2, +\tau/2)$.

The Quasi-Normal Approximation (Lesieur, 1997, Chap VII-2, QNA hereafter) is adopted so that the fourth-order velocity correlations can be written as a product of second-order velocity correlations $\langle u'_i u'_j \rangle$. The properties of $\langle u'_i u'_j \rangle$ are well known in the Fourier domain (\vec{k}, ω) where \vec{k} and ω are the wavenumber and the frequency associated with a turbulent element. The second-order velocity correlations can be expressed formally in term of $E(k, \omega)$ the turbulent kinetic energy spectrum. Following Stein (1967), the velocity energy spectrum $E(k, \omega)$ is written as

$$E(k, \omega) = E(k) \chi_k(\omega) \quad (2.9)$$

where $\chi_k(\omega)$ is a frequency-dependent factor which can be related to the time correlation of the eddies in the frequency space. Finally a general expression for C_R^2 involving $E(k)$ and $\chi_k(\omega)$ is given in Paper I:

$$C_R^2 = 4\pi^3 \mathcal{G} \int_0^M dm \rho_0 \left(\frac{d\xi_r}{dr} \right)^2 \int_0^\infty dk \frac{E^2(k)}{k^2} \chi_k(\omega_0) \quad (2.10)$$

where \mathcal{G} is a constant anisotropic factor.

2.2 Contribution of entropy fluctuations

We assume in Paper I that δs_t and s_t in Eq.(2.3) act as passive scalars. Let $E_s(k, \omega)$ be the spectrum of the scalar correlation product $\langle s'_t s''_t \rangle$. It is also assumed that the frequency-dependent component of $\langle s'_t s''_t \rangle$ is the same than those of the velocity field correlation product $\langle u'_i u'_j \rangle$ and that $E_s(k, \omega)$ can be decomposed as $E(k, \omega)$ in Eq.(2.9).

As a consequence of the above assumptions, it is then demonstrated that the correlation product of the Lagrangian entropy fluctuations $\langle \delta s'_t \delta s''_t \rangle$ vanishes after integration over \vec{r} in Eq.(2.8). This result may be explained as follows: turbulence and oscillation are coupled through the phase term $e^{-i\omega_0\tau}$ and through the turbulent time spectrum $\chi_k(\omega)$. Therefore this coupling occurs at frequencies close to the oscillation frequency ω_0 . It then follows that the coupling between turbulence and oscillation involves eddies of wavenumber $k \gg k_{osc}$ where k_{osc} is the wavenumber of the mode and k that of an eddy. On the other hand the spatial component of $\langle \delta s'_t \delta s''_t \rangle$ in the Fourier space favors eddies with the largest size ($k \rightarrow 0$). These two opposite effects clearly are incompatible and lead to a vanishing contribution for the Lagrangian entropy fluctuation.

This does not happen for the contribution of the Reynolds source term which involves the fourth-order velocity correlation product. According to the QNA this term can be decomposed in term of a product of two second-order velocity correlations. Coupling with the oscillation then becomes non-linear and leads to an effective non zero contribution. Thus *only non-linear terms, with respect to the fluctuations, can contribute to mode excitation while linear terms do not*. This may be considered as a general result.

With an analogous procedure than for the Reynolds stress contribution, we show that the entropy contribution can be expressed as:

$$C_S^2 = \frac{4\pi^3 \mathcal{H}}{\omega_0^2} \int d^3 x_0 \left(\alpha_s \frac{d\xi_r}{dr} \right)^2 g_r \int dk \frac{E_s(k) E(k)}{k^2} \int_{-\infty}^{+\infty} d\omega \chi_k(\omega_0 + \omega) \chi_k(\omega) \quad (2.11)$$

with $g_r(\vec{\xi}_r, r)$ a function which expression is given in Paper I and \mathcal{H} a constant anisotropic factor similar to \mathcal{G} .

3 Results in the Solar case

3.1 Models for the solar turbulence

On one hand, the turbulence theory tells us that $E(k)$ follows the Kolmogorov spectrum as $E(k) \propto k^p$ with the slope $p = -5/3$. On the other hand, observations of the solar granulation allow one to determine the turbulent kinetic spectrum $E(k)$ of the Sun.

Several kinetic turbulent spectra have been suggested by different observations of the solar granulation: the “Raised Kolmogorov Spectrum” (RKS hereafter), the “Nesis Kolmogorov Spectrum” (NKS hereafter) and the “Broad Kolmogorov Spectrum” (BKS hereafter). These spectra (Figure 1) obey the Kolmogorov law for $k \geq k_0$ where k_0 is the wavenumber at which the turbulent cascade begins. They differ in the injection region ($k < k_0$) where they are characterized by different values for the slope p .

The wavenumber k_0 is unknown but can be related to the mixing length Λ as $k_0 = 2\pi / (\beta\Lambda)$ where β is a free parameter introduced for the arbitrariness of such definition (Paper I). Moreover the definition of the eddy correlation time, which enters the description of the turbulent excitation, is somewhat arbitrary and is therefore gauged by introducing an additional free parameter λ .

3.2 Oscillation power

The power injected into the solar oscillations is related to the rms value v_s of the surface velocity as $v_s^2(\omega_0) = \xi_r^2(r_s) P(\omega_0) / 2\eta I$ where r_s is the radius at which oscillations are measured. Observations of the solar oscillations provide v_s and η such that $P(\omega_0)$ can be evaluated from observations. On the theoretical side, the power $P(\omega_0)$ is computed according to Eqs(2.6, 2.6, 2.10, 2.11) for a calibrated solar model. The physical ingredients of the model are detailed in Samadi et al. (2001, Paper II hereafter). As was done in Paper II, the free parameters (β, λ) are adjusted in order to obtain the best fit to the solar seismic observations by Libbrecht (1988).

Theoretical arguments support the fact that the current theory of stochastic excitation is less justified for the low frequency modes than for the high frequency ones (see below section 4.1). Therefore in the following β and λ are adjusted in order to reproduce, as best as possible, the frequency dependency of $v_s(\omega_0)$ at high frequencies ($\nu \gtrsim 3.5$ mHz).

Results of the fitting are shown in Fig. 1. As a consequence of the adjustment, all the kinetic turbulent spectra fit well the solar observations at high frequency ($\nu \gtrsim 3.5$ mHz) while the main differences are observed at low frequency. The overall best agreement is obtained with the NKS.

Moreover we find that the entropy source term significantly contributes to the excitation process of solar-like oscillations by turbulent convection, in agreement with the results of GMK and simulations by Stein and Nordlund (1991).

4 Some open issues

4.1 Static and dynamic properties of the turbulent medium

As stressed by Rieutord et al. (2000), differences at low wavenumber between the kinetic spectra obtained from different observations of the solar granulation are a consequence of uncontrolled data-averaging procedures. Mean average (static) properties of the kinetic spectrum at low wavenumber (mesogranulation) are thus not yet well represented by the observations of the solar surface. In the same way, Nordlund et al. (1997) demonstrated with a numerical simulation that observations of the solar granulation cannot provide a meaningful determination of the turbulent properties. Therefore, the mean average (static) properties of the different turbulent spectra - suggested by the current observations of the solar granulation - are not yet well established.

On the theoretical side, the derivation of Eq. (2.10) and Eq.(2.11) uses the decomposition of Eq.(2.9). $E(k)$ depicts the static properties of the turbulent medium whereas $E(k, \omega)$ characterizes the dynamic properties. In Paper II different forms for $\chi_k(\omega)$ - which were suggested by Musielak et al. (1994) - were tested. From the comparison with the solar seismic data we cannot clearly discriminate between the different forms and we finally adopted the gaussian form (Paper II).

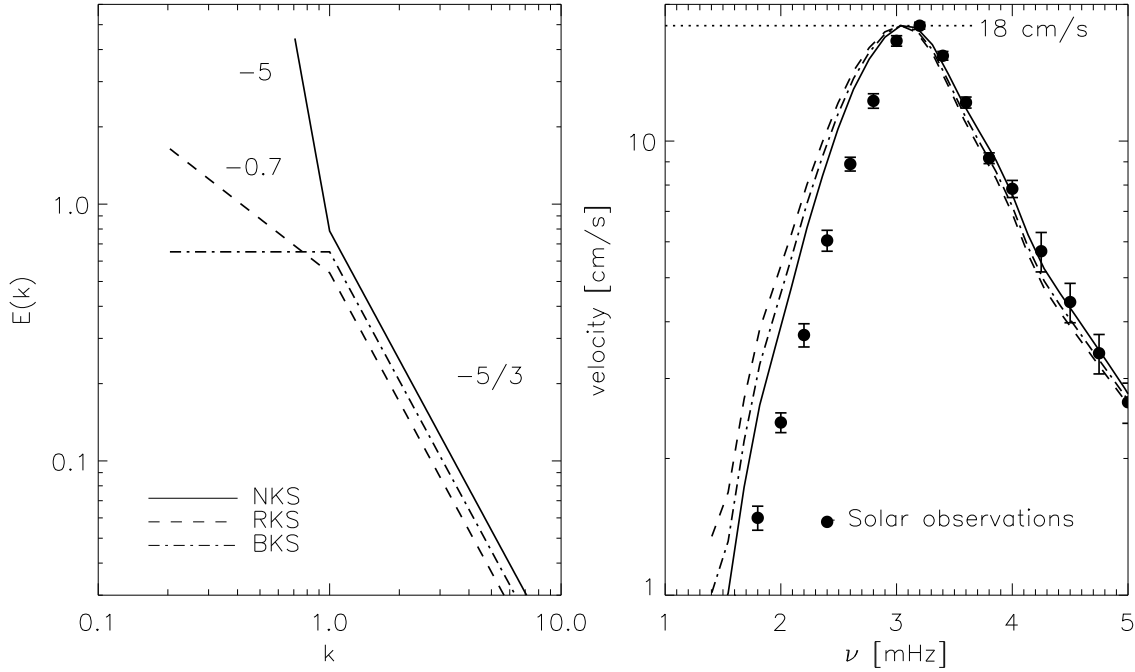


Fig. 1. Left panel: Kinetic turbulent spectra versus wavenumber k . **Right panel:** Computed surface velocity v_s assuming the turbulent spectra plotted on the left panel. λ and β values result from fitting the computed v_s to the solar seismic observations by Libbrecht (1988) using the observed damping rate η .

However it is suggested in Samadi (2000) that the decomposition of Eq.(2.9) is not valid at small wavenumber (in the injection region, i.e. for $k \lesssim k_0$). Indeed the high frequency modes involve small size turbulent elements ($k > k_0$). At this scale (the inertial range) the lifetime τ_k of the turbulent elements are shorter than the timescale τ_Λ at which energy is injected into the turbulent cascade. Therefore at small scales injection of energy into the cascade appears stationary and the dynamic properties of $E(k, \omega)$ are only fixed by the lifetime τ_k of the eddies. The line-width of the function $\chi_k(\omega)$ provides a statistical measure of τ_k . As a consequence, the decomposition of Eq.(2.9) should be valid for $k \gtrsim k_0$.

In contrast at large scales, ($k \lesssim k_0$), the lifetime of the turbulent elements is of the same order than τ_Λ . Therefore the energy transfer between large and small elements is no longer stationary and consequently the decomposition of Eq.(2.9) is probably no longer valid.

This statement explains the large changes observed at low frequency between the different spectra (Fig. 1). Indeed the low frequency modes involve the largest eddies and the changes at low frequency are therefore due to the differences in the static properties of the turbulent spectra in the injection region (see Fig. 1). The best agreement is obtained with the NKS which however appears as the less realistic spectrum. In contrast the more plausible spectra (RKS and BKS) significantly overestimate the amplitudes at low frequency. These results prove that the dynamic properties of $E(k, \omega)$ is not correctly modeled through the decomposition of Eq.(2.9) in the range $k \lesssim k_0$. Moreover, the decomposition of Eq.(2.9) is also used for the turbulent spectrum $E_s(k, \omega)$ of the entropy fluctuations. Therefore the above conclusions also apply to $E_s(k, \omega)$.

4.2 Excitation of the non-radial oscillations

The present theory of stochastic excitation only concerns the radial p -modes. It is shown in Samadi (2000) that the stratification does not affect the transfer of acoustic power into the radial modes.

The high ℓ degree modes propagate in both radial and horizontal directions. The acoustic power injected into high ℓ degree modes is therefore more sensitive to the stratification.

The current theory is mainly based on the approximation that the stratification and the sources of excitation are well decoupled. Stein (1967) showed that the acoustic emission arising from the stratification is negligible compared to the Reynolds stress emission. However some high ℓ degree modes propagate mainly in the horizontal direction in the excitation region. For such modes it is important to evaluate the contribution of the stratification to the acoustic power emission.

5 Conclusion

The present paper summarizes the theory of solar oscillation excitation as it is exposed in Samadi and Goupil (2001). In particular assumptions and approximations which were used are recalled here with some details. Open issues in this field are also discussed such as the way the turbulence is modeled and the case of non-radial modes:

The way the turbulence is currently modeled at large scale length introduces large uncertainties in the computation of the power injected into the low frequency modes ($\nu \lesssim 3.3$ mHz in the solar case). A more robust modeling of the large scales is thus needed in order to extend to low frequencies the validity of the current theory. This can be achieved with the use of 3D simulations of the solar convection zone (work in progress).

Current theories only concern the excitation of radial modes. For non-radial modes effect of the stratification upon the excitation may be important whereas it is negligible for radial ones. Therefore effects of the stratification to the power emission in the high ℓ degree modes, as well as the ℓ dependency of the current theory of stochastic excitation, should be addressed (work in progress).

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